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P. G. Demidov Yaroslavl State University

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ALGEBRAIC TOPOLOGY AND ABELIAN FUNCTIONS

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ABSTRACTS

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Plenary lectures

Generalizations of the Davis–Januszkiewicz construction

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Briefly, a toric manifold M^{2n} is a manifold covered by local charts $\mathbb{C}^n$, each with the standard action of a real n-dimensional torus T^n, compatible in such a way that the quotient M^{2n}/T^n has the structure of a *simple* polytope P^n. The construction of Davis and Januszkiewicz [4, Section 1.5] realizes all toric manifolds and, in particular, all smooth projective toric varieties. The key ingredient is a *characteristic function*

$$\lambda : \mathcal{F} \longrightarrow \mathbb{Z}^n \tag{1}$$

from the set of facets of the polytope P^n into an n-dimensional integer lattice, satisfying certain conditions. The construction is

$$M^{2n} \cong M^{2n}(\lambda) = T^n \times P^n/\sim \tag{2}$$

where the equivalence relation $\sim$ is defined in terms of the function λ. Associated to P^n is a simplicial complex $K(P)$ with m

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vertices corresponding to the m facets of P^n. In the *moment-angle complex* formalism established by Buchstaber and Panov [3], the map λ determines a subtorus $\ker(\lambda) = T^{m-n} \subset T^m$ and a homeomorphism

$$T^n \times P^n \big/ \sim \;\cong\; Z(K_P; (D^2, S^1)) \Big/ T^{m-n}. \qquad (3)$$

A reinterpretation of this construction allows for a generalization in such a way that it can be used to construct the family of manifolds $M(J)$ described in [2]. These are determined by a sequence of positive integers $J = (j_1, j_2, \ldots, j_m)$. Associated to the new construction are polyhedral products $Z(K_P; (\mathbb{C}P^\infty, \mathbb{C}P^{J-1}))$ that play the role which the spaces $Z(K_P; (\mathbb{C}P^\infty, *))$ do in the standard case. The construction is generalized further to yield spaces associated to the *composed* simplicial complexes of Ayzenberg [1].

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Periods of second kind differentials of (n, s)-curves

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The following problem is discussed and solved in particular cases here: *Given curve $\mathcal{C}$ of genus $g > 1$ and its $\mathfrak{a}$ and $\mathfrak{b}$-periods of holomorphic differentials $2\omega, 2\omega'$. Let $2\eta, 2\eta'$ are periods of the second kind differentials conjugated to $2\omega, 2\omega'$ according to the generalized Legendre relations,*

$$\eta^T\omega = \omega^T\eta, \quad \eta^T\omega' - \omega^T\eta' = \frac{\imath\pi}{2}, \quad {\eta'}^T\omega' = {\omega'}^T\eta'.$$

Express periods of the second kind differentials in terms these data including θ-constants, depending on Riemann period matrix $\tau = \omega'/\omega$ and coefficients of polynomial defining the curve $\mathcal{C}$

In the case of elliptic curve $y^2 = 4x^3 - g_2x - g_3$ this question is answered by the Weierstrass formulae

$$\eta = -\frac{1}{12\omega}\sum_{k=2}^{4}\frac{\vartheta_k''(0)}{\vartheta_k(0)} \quad \text{and equivalently} \quad \eta = -\frac{1}{12\omega}\frac{\vartheta_1'''(0)}{\vartheta_1'(0)}.$$

The θ-constant representation of periods of the second kind differentials is important for defining of the multi-dimensional σ-function which theory attract many attention now and was intensively developed during the last decade by V.M.Buchstaber with co-workers, see the recent book project V. M. Buchstaber and V. Z. Enolski and D. V. Leykin, *Multi-Dimensional Sigma-Functions*, arXiv:1208.0990 [math-ph], 267, pp. 2012.

We present here an approach to solve the formulated above problem for the family of (n, s)-curves that represent a natural generalization of Weierstrass elliptic cubic to higher genera,

$$f(x,y) = y^n - x^s - \sum_{\alpha,\beta} \lambda_{\alpha n+\beta s} x^\alpha y^\beta$$

with $\lambda_k \in \mathbb{C}$ and $0 \le \alpha < s-1, \quad , 0 \le \beta < n-1$. The case of hyperelliphic curves and especially genus two curve were considered in details. A number of new (to the best knowledge of the authors) θ-constant relations is derived as consequence of the formulae obtained and a generalization of the Jacobi derivative formula is one from them.

Zero modes in Liouville model

Ludwig D. Faddeev *(Saint-Petersburg Department of Steklov Mathematical Institute of RAS, Russia)*

We show that the monodromy matrix elements for the Lax operator are natural zero modes in the Liouville model. Both classical and quantum models are considered. We also describe the evolution operator for discrete time intervals.

Singular fibrations with fibers $(S^n)^k$

Tadeusz Januszkiewicz *(Instytut Matematyczny PAN, Poland)*

Toric spaces can be thought of as simplest possible torus fibrations with simplest possible degenerations of fibers (at least from the point of vie of the group actions — algebraic geometers have a different idea about the “simplest possible degenerations” of Abelian varieties).

Many years ago Rick Scott, then a PhD sudent of Robert MacPherson, proposed a generalization of toric spaces over a polygon as simplest possible $S^3 \times S^3$ fibrations with simplest possible degenerations of fibers.

In my talk I will try to recall the concept, its links with classical differential topology, similarities and differences with toric spaces and highlight some challenges.

Co-convex bodies and multiplicities

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I. Given a closed strictly convex cone $C \subset \mathbb{R}^n$, a theory of C-co-convex bodies can be developed [1]. It can be thought of as a local version of the usual theory of convex bodies in convex geometry. A closed bounded subset $\Delta \subset C$ is *C-co-convex* if the complement $\Delta^* = C \setminus \Delta$ is an (unbounded) convex set. A *linear combination* $\sum \lambda_i \Delta_i$, where $\lambda_i \geq 0$, of C-co-convex bodies Δ_i is by definition the complement $C \setminus \sum \lambda_i \Delta_i^*$ of the linear combination (in Minkowski sense) of the convex sets Δ_i^*. The volume of a linear combination of C-co-convex bodies is a degree n homogenous polynomial in the λ_i. Thus one can define the *mixed volume* of C-co-convex bodies. We prove a version of the *Brunn–Minkowsky inequality* $V^{1/n}(\Delta_1+\Delta_2) \leq V^{1/n}(\Delta_1)+V^{1/n}(\Delta_2)$ and a version of the *Alexandrov–Fenchel inequality*

$$V(\Delta_1, \Delta_1, \dots, \Delta_n)V(\Delta_2, \Delta_2, \dots, \Delta_n) \geq V^2(\Delta_1, \Delta_2, \dots, \Delta_n))$$

for volumes and mixed volumes of C-co-convex bodies [1].

II. In [2], to an $\mathfrak{m}$-*primary ideal I in an n-dimensional regular local ring* we associate a C-co-convex body, where $C = \mathbb{R}^n_{\geq 0}$ is the positive octant in $\mathbb{R}^n$, which encodes information about the Samuel multiplicity $e(I)$ of I. More generally, we associate a C-co-convex body, where C is the cone associated to an *affine toric variety*, to an $\mathfrak{m}$-primary ideal in the local ring of this variety. This construction is in the spirit of the theory of Gröbner bases and Newton polyhedra on one hand, and the theory of Newton–Okounkov bodies for linear systems on the other hand. Using this we obtain a new proof of a Brunn-Minkowski type inequality $e(IJ)^{1/n} \leq e(I)^{1/n} + e(J)^{1/n}$ for multiplicities of ideals due to

Teissier and Rees–Sharp. We also obtain a simple proof of the local Bernstein–Koushnirenko formula for toric varieties.

III. Our construction in [2] works for local rings of toroidal singularities and not for general local rings. Thus this approach does not give a proof of an *Alexandrov–Fenchel inequality*

$$e(I_1, I_1, I_3, \ldots, I_n)e(I_2, I_2, I_3, \ldots, I_n) \geq e(I_1, I_2, I_3, \ldots, I_n)^2,$$

for mixed multiplicities of ideals. But in [3] we show that one can reduce all the properties of mixed multiplicities of $\mathfrak{m}$-primary ideals in general local rings (including the Hilbert–Samuel formula) to the (global) intersection theory of linear subspaces of rational functions developed in [3]. In [4] we prove the local algebraic Alexandrov–Fenchel inequality in the same way as we did it for the global case in [3]. From this we then obtain the geometric inequality in [1] from the algebraic one [4] (in a similar fashion that the classical Alelexandrov–Fenchel inequality was deduced from it's algebraic analog in [3]).

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The universal Whitham hierarchy and geometry of moduli spaces of curves with punctures

Igor M. Krichever *(Columbia University, USA)*

The moduli spaces of algebraic curves with a pair of meromorphic differentials is central for the Hamiltonian theory of integrable systems, Whitham perturbation theory of soliton equations, quantum topological field theories, Sieberg–Witten solution of $N = 2$ SUSY gauge theories and Laplacian growth problems. In the talk their applications to a study of geometry of moduli spaces of curves with punctures will be presented.

Toric origami manifolds in toric topology

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A *symplectic toric* (or *toric symplectic*) manifold is a compact connected symplectic manifold with an effective Hamiltonian action of a torus T with $\dim T = \frac{1}{2}\dim M$. A famous theorem by Delzant says that the correspondence from symplectic toric manifolds to simple convex polytopes called *Delzant polytopes* via moment maps is bijective. It implies that a symplectic toric manifold is equivariantly diffeomorphic to a projective smooth toric variety and vice versa.

A *folded symplectic form* on a $2n$-dimensional manifold M is a closed 2-form ω whose top power ω^n vanishes transversally on a subset Z and whose restriction to points in Z has maximal rank. Then Z is a codimension-1 submanifold of M and called the *fold*. Using the h-principle, Cannas da Silva [1] shows that an orientable compact manifold admits a folded symplectic form if and only if it admits a stably almost complex structure.

The maximality of the restriction of ω to Z implies the existence of a line field on Z and ω is called an *origami form* if the line filed is the vertical bundle of some principal S^1-fibration $Z \to B$. The notions of a Hamiltonian action and a moment map can be defined similarly to the symplectic case, and a *toric origami manifold* is defined in [2] to be a compact connected origami manifold (M, ω) equipped with an effective Hamiltonian action of a torus T with $\dim T = \frac{1}{2}\dim M$. The sphere S^{2n} with the standard action of T can be a toric origami manifold while

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it cannot be a symplectic toric manifold when $n \geq 2$. A toric origami manifold M is not necessary simply connected; it can be even non-orientable. The cohomology of a toric origami manifold M is investigated by Holm and Pires [4] when M is simply connected (equivalently when M/T is contractible).

Cannas da Silva, Guillemin and Pires [2] show that toric origami manifolds bijectively correspond to origami templates via moment maps. An *origami template* is a collection of Delzant polytopes with some folding data. Like a fan is associated to a Delzant polytope, a multi-fan can be associated to an origami template. A *multi-fan* is a collection of cones satisfying certain conditions, where cones may overlap unlike an ordinary fan (see [3]).

In this talk, we discuss toric origami manifolds from the viewpoint of toric topology. This talk is based on a joint work with Seonjeong Park ([5]).

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A proof of the geometric case of a conjecture of Grothendieck and Serre concerning principal bundles

Ivan A. Panin *(Saint-Petersburg Department of Steklov Mathematical Institute of RAS, Russia)*

This talk is about our joint work with Roman Fedorov. Assume that U is a regular scheme, G is a reductive U-group scheme, and $\mathcal{G}$ is a principal G-bundle. It is well known that such a bundle is trivial locally in étale topology but in general not in Zariski topology. A. Grothendieck and J.-P. Serre conjectured that $\mathcal{G}$ is trivial locally in Zariski topology, if it is trivial at all the generic points. We proved of this conjecture for regular local rings R, containing infinite fields. Our proof was inspired by the theory of affine Grassmannians. It is also based significantly on the geometric part of a paper of the second author with A. Stavrova and N. Vavilov.

Thom complexes and stable decompositions

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Since the 1940s it has become apparent that cartesian products of the form $X \times \mathbb{R}^n$, and certain generalisations, may simplify and illuminate the study of a topological space X. For example, smoothings of a PL-manifold M correspond to those of $M \times \mathbb{R}^n$ in an appropriate sense, for any $n \geq 0$; this fact extends to non-trivial vector bundles over M, and ensures that smoothings are classified by lifts of its tangent microbundle. So far as homotopy theory is concerned, the total space of a vector bundle α over X is most naturally replaced by the Thom complex $Th(\alpha)$; this may be interpreted as a type of compactification of the total space of α. Thom's construction actually has its roots in work of Pontryagin, who pioneered the concept for trivial α.

During the 1940s and 50s, Pontryagin and Thom applied their ideas to the development of framed and unoriented cobordism theories respectively. This theme continued into the 1960s, as Milnor and Novikov established the foundations of the complex analogue, otherwise known as MU-theory; its derivatives are still amongst the most powerful tools of stable homotopy. In the course of Victor Buchstaber's career, the scope and influence of Thom complexes have increased remarkably in breadth and depth. Over those four and a half decades, homotopy theorists have learned how to decompose many spaces that arise from complex geometry into smaller building blocks, which are often Thom complexes of non-trivial vector bundles. Conversely, certain Thom complexes have themselves been decomposed into simpler blocks, some of which are also Thom complexes of a more fundamental nature.

In this talk I shall survey a few representative samples of such decompositions, and hope to give the flavour of the subject in a style that is accessible to non-experts. I plan to cover as many of the following as time permits:

- Snaith's stable splitting of infinite Grassmannians $BU(n)$ into wedges of Thom spaces $MU(k)$, and Mitchell and Richter's refinement over $\Omega SU(n)$;

- Mimura, Nishida, and Toda's splitting of $\mathbb{C}P^\infty$ into $p-1$ blocks ($p-2$ of which are Thom spaces) after localisation at a prime p;

- More recent examples involving families of toric varieties, such as Bott towers and weighted projective spaces.

In particular, I hope to discuss how straightforward calculations in homology and cohomology may be used to provide evidence for (and against!) the existence of splittings, and the possibility that certain blocks might be realised as Thom complexes.

Classification of complex projective towers up to dimension 8 and cohomological rigidity

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Shintaro Kuroki *(University of Toronto, Canada),* Shintaro.kuroki@utoronto.ca

A complex projective tower or simply a $\mathbb{C}P$-tower is an iterated complex projective fibrations starting from a point. In this paper we classify all 6-dimensional $\mathbb{C}P$-towers up to diffeomorphism, and as a consequence, we show that all such manifolds are cohomologically rigid, i.e., they are completely determined up to diffeomorphism by their cohomology rings. We also show that cohomological rigidity is not valid for 8-dimensional $\mathbb{C}P$-towers by classifying some $\mathbb{C}P^1$-fibrations over $\mathbb{C}P^3$ up to diffeomorphism. As a corollary we show that such $\mathbb{C}P$-towers are diffeomorphic if they are homotopy equivalent.

Elliptic solitons and minimal tori with planar ends

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We expose our joint results with C. Bohle on spectral curves of elliptic solitons which appear as spectral curves of minimal tori with planar ends in the Euclidean 3-space [1, 2].

In particular, it is shown that

- there exist two-dimensional Dirac operators with double-periodic potentials and reducible spectral curves on the zero energy level;
- one can define a spectral curve for the Cauchy–Riemann operator on a punctured elliptic curve if one imposes appropriate boundary conditions. Algebraic curves of the type thus obtained appear as irreducible components of spectral curves of minimal tori with planar ends in $\mathbb{R}^3$ and coincide with the spectral curves of certain elliptic KP solitons.

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Dynamics of metrics and measures: new and old problems

Anatoly M. Vershik *(Saint-Petersburg Department of Steklov Mathematical Institute of RAS, Russia)*

- Dynamics of the metrics on the spaces with invariant measures. Scaling entropy of the action is a generalization of the topological and Kolmogorov–Sinai entropy. The known and open problems. Examples.

- Classification of the metric triples and structural problems. Matrix distributions and classification of the functions. Invariant distribibution of random matrices.

- Virtual continuity and the theorems about traces.

Elliptic Dunkl operators and Calogero–Moser systems

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In 1994 Buchstaber, Felder and the speaker [1] described the general form of the commuting Dunkl operators. The problem was reduced to the solution of certain functional equations, which were explicitly solved in terms of elliptic functions.

However, since the elliptic Dunkl operators turned out to be not invariant under permutations, the question of how to use them to construct the integrals of quantum elliptic Calogero–Moser problem remained open. An interesting idea suggested by the authors of [1] was to replace in Heckman's approach the symmetric polynomials by the integrals of the classical Calogero–Moser system in spectral parameters.

In the talk I will explain in more detail this construction, which has recently been fully justified in [2].

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On highly regular embeddings

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A *k-regular embedding* $X \to \mathbb{R}^N$ maps any k pairwise distinct points in a topological space X to k linearly independent vectors.

The study of the existence of k-regular maps was initiated by Borsuk [2] in 1957 and latter attracted additional attention due to its connection with approximation theory. The problem and its extensions were extensively studied by Chisholm, Cohen, Handel, and others in the 1970's and 1980's, and then again by Handel and Vassiliev in the 1990.

Our main result on k-regular maps is the following lower bound for the existence of such maps between Euclidean spaces, in which $\alpha(k)$ denotes the number of ones in the dyadic expansion of k:

Theorem 1 *For any $d \geq 1$ and $k \geq 1$ there is no k-regular map $\mathbb{R}^d \to \mathbb{R}^N$ for $N < d(k - \alpha(k)) + \alpha(k)$.*

This reproduces a result of Chisholm [3] from 1979 for the case of d being a power of 2; for the other values of d our bounds are in general better than Karasev's [7], who had only recently gone beyond Chisholm's special case. In particular, our lower bound turns out to be tight for $k = 3$.

The framework of Cohen & Handel [4] relates the existence of a k-regular map to the existence of a specific inverse of an

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appropriate vector bundle. Thus non-existence of regular maps into $\mathbb{R}^N$ for small N follows from the non-vanising of specific dual Stiefel–Whitney classes. This we prove using the general Borsuk–Ulam–Bourgin–Yang theorem combined with a key observation by Hung [6] about the cohomology algebras of configuration spaces.

Our study [1] produces similar topological lower bound results also for the existence of *ℓ-skew embeddings* $\mathbb{R}^d \to \mathbb{R}^N$ for which we require that the images of the tangent spaces of any ℓ distinct points are skew affine subspaces. This extends work by Ghomi & Tabachnikov for $\ell = 2$.

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Invited lectures

Cox rings, universal torsors, and infinite transitivity

Ivan V. Arzhantsev *(Lomonosov Moscow State University, Russia),* `arjantse@mccme.ru`

The aim of the talk is to introduce an important invariant of an algebraic variety – the total coordinate ring, or the Cox ring [5], – and to describe its applications in algebraic geometry and the theory of transformation groups [6], [1]. Cox rings possess nice algebraic properties and define a canonical quotient presentation of a variety related to the construction of universal torsor from arithmetic geometry [7].

Also we will discuss recent results on infinite transitivity of the action of automorphism groups on affine varieties [2], [3] and universal torsors [4]. More precisely, let us define the special automorphism group $\mathrm{SAut}(X)$ of an algebraic variety X as the subgroup of $\mathrm{Aut}(X)$ generated by all one-parameter unipotent subgroups. Let X be an affine variety X of dimension at least 2. The main result of [2] states that if the group $\mathrm{SAut}(X)$ is transitive on the smooth locus X_{reg}, then it is infinitely transitive on X_{reg}. In turn, the transitivity is equivalent to the flexibility of X. The latter means that for every smooth point x on X the tangent space T_xX is spanned by the velocity vectors at x of one-parameter unipotent subgroups. Examples of flexible varieties are non-degenerate affine toric varieties, suspensions and smooth affine varieties with a generically transitive action of a semisimple group.

Let X be an algebraic variety covered by open charts isomorphic to the affine space and $q : \widehat{X} \to X$ be the universal torsor over X. We prove that the special automorphism group $\mathrm{SAut}(\widehat{X})$ of the quasiaffine variety $\widehat{X}$ acts on $\widehat{X}$ infinitely transitively [4].

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Some topological problems in systolic geometry

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Let G be a finitely representable group, and $\mathbf{a} \in H_m(G, \mathbb{Z})$ a non trivial homology class of dimension $m \geq 3$. A *geometric cycle* (X, f) *representing certain class* $\mathbf{a}$ is a pair (X, f) consisting of an orientable (connected) pseudomanifold X of dimension m and continuous map $f : X \longrightarrow K(G, 1)$ such that $f_*([X]) = \mathbf{a}$ where $[X]$ denotes the fundamental class of X and $K(G, 1)$ is the Eilenberg-McLane space corresponding to G. Furthermore the representation is said to be *normal* if the induced map $f_\sharp : \pi_1(X) \longrightarrow G$ is an epimorphism. Given a geometric cycle (X, f), then for any polyhedral metric g on X we can consider the *relative homotopic systole* denoted by $\mathrm{sys}_f(X, g)$ and defined as the least length of a loop γ of X whose image under f is not contractible. So the *systolic volume* of the geometric cycle (X, f) is the value

$$\sigma_f(X) := \inf_g \frac{\mathrm{Vol}(X, g)}{\mathrm{sys}_f(X, g)^m},$$

where the infimum is taken over all polyhedral metrics g on X and $\mathrm{Vol}(X, g)$ denotes the m-dimensional volume of X.

From [1], we have $\sigma_m := \inf_g \sigma_f(X) > 0$, for any $m \geq 1$, the infimum being taken over all geometric cycles (X, f) representing a non trivial homology class of dimension m. Following [1] we define the systolic volume of the pair $(G, \mathbf{a})$ as the number $\sigma(G, \mathbf{a}) := \inf_{(X,g)} \sigma_f(X)$, where the infimum is taken over all geometric cycles (X, f) representing the class $\mathbf{a}$.

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Any integer class is representable by a geometric cycle. The systolic volume of $(G, \mathbf{a})$ is thus well defined and satisfies $\sigma(G, \mathbf{a}) \geq \sigma_m$. But it is not clear if the infimum value $(G, \mathbf{a})$ is actually a minimum and what is the structure of a geometric cycle that might achieve it. A pseudomanifold is said *admissible* if any element of the fundamental group can be represented by a curve not going throuhg the singular locus of X.

Theorem 1 ([2]) *Let G be a finitely presentable group and $\mathbf{a} \in H_m(G, \mathbb{Z})$ a homology class of dimension $m \geq 3$. For any normal representation of $\mathbf{a}$ by an admissible geometric cycle (X, f),*

$$\sigma(G, \mathbf{a}) = \sigma_f(X).$$

Furthermore, there always exists a normal representation of $\mathbf{a}$ by an admissible geometric cycle.

Thus the infimum in the definition of the systolic volume of a homology class is a minimum. It can be shown that the normalization condition (that is $f_\sharp$ is an epimorphism between fundamental groups) can not be relaxed in the theorem.

So $\sigma(G, \mathbf{a})$ is a well defined function in two variables: a finitely presentable group G and a class $\mathbf{a}$ in its own homology. This function possesses a number of interesting properties which will be observed in the talk and a number of open questions will be discussed there as well.

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On some rational and integral complex genera

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The general four variable complex elliptic genus ϕ_{KH} on $MU_* \otimes \mathbb{Q}$ is defined [5],[6] to be determined by the following property: if one denotes by $f_{Kr}(x)$ the exponent of the Krichever [2], [3], [6] universal formal group law $\mathcal{F}_{Kr}$, then the series $h(x) := f'_{Kr}(x)/f_{Kr}(x)$ satisfies the differential equation $(h'(x))^2 = S(h(x))$, where $S(x) = x^4 + p_1x^3 + p_2x^2 + p_2x + p_4$, the generic monic polynomial of degree 4 with formal parameters p_i of weights $|p_i| = 2i$.

To generalize naturally the Oshanin's elliptic genus from $\Omega_*^{SO} \otimes \mathbb{Q}$ to $\mathbb{Q}[\mu, \epsilon]$ [8], new elliptic genus ψ is defined in [9] to be the genus $\psi : MU_* \otimes \mathbb{Q} \to \mathbb{Q}[p_1, p_2, p_3, p_4]$ whose logarithm equals

$$\int_0^x \frac{dt}{\omega(t)}, \quad \omega(t) = \sqrt{1 + p_1t + p_2t^2 + p_2t^3 + p_4t^4}$$

and p_i are again formal parameters $|p_i| = 2i$.

Clearly to calculate the values of ψ on CP_i, the generators of the rational complex bordism ring [7] $MU_* \otimes \mathbb{Q} = \mathbb{Q}[CP_1, CP_2, \dots]$ we need only the Taylor expansion of $(1+y)^{-1/2}$ by above definition as $log'_\psi = \sum_{i \geq 1} \psi(CP_i)x^i$.

It is natural to ask if there exists similar elementary way, different from formulas in [4], for calculation of ϕ_{KH}. Our answer is the formula

$$\psi \circ \kappa^{-1},$$

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where ψ is the genus mentioned above and κ classifies the formal group law strictly isomorphic [7] to the universal formal group law under strict isomorphism $\nu(x) = xCP(x)$. Thus $\kappa(CP_i)$ is determined by equating coefficients at x^i in $\sum_{i\geq 1} \frac{\kappa(CP_i)}{i+1} x^{i+1} = \sum_{i\geq 1} \frac{CP_i}{i+1}(\nu^{-1}(x))^{i+1}$.

We construct certain elements A_{ij} in the Lazard ring and give an alternative definition of the universal Buchstaber [2] formal group law. This implies that universal Buchstaber and Krichever formal groups coincide and the corresponding coefficient ring is the quotient of the Lazard ring by the ideal generated by all A_{ij}, $i, j \geq 3$.

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On G_2 Holonomy Riemannian Metrics on Deformations of Cones over $S^3 \times S^3$

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Consider a deformation of standard conic metric over the space $M = S^3 \times S^3$. This conic metric can be written as

$$d\bar{s}^2 = dt^2 + \sum_{i=1}^{3} A_i(t)^2 \left(\eta_i + \tilde{\eta}_i\right)^2 + \sum_{i=1}^{3} B_i(t)^2 \left(\eta_i - \tilde{\eta}_i\right)^2,$$

where $\eta_i, \tilde{\eta}_i$ is the standard coframe of 1-forms, whereas the functions $A_i(t), B_i(t)$ define a deformation of the cone singularity. In the paper [1] a system of differential equations is written down, which guarantees that the metric $d\bar{s}^2$ has the holonomy group containing in G_2 and a particular solution of this system is found. This particular solution corresponds to a metric with the holonomy group G_2 on $S^3 \times \mathbb{R}^4$. Another particular solution with the same topology was found in [2]. In the proposed work we continue to study this class of metrics, while setting $A_2 = A_3$, $B_2 = B_3$ and considering boundary conditions different from that from [1, 2]. This yields the metrics with a different topological structure. Namely, we require that at the vertex of the cone only the function B_1 turns to zero. This results in that the Riemannian metric $d\bar{s}^2$ is defined on $H^4 \times S^3$, where H is the space of canonical complex linear bundle over S^2, where H^4 is its fourth tensor power. The main result is formulated in the following theorem:

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Theorem 1 *There exists a one-parametric family of mutually non-homothetic complete Riemannian metrics of the form* $d\bar{s}^2$ *with holonomy group* G_2 *on* $H^4 \times S^3$*, whereas the metrics can be parameterized by the set of initial data* $(A_1(0), A_2(0), B_1(0), B_2(0)) = (\mu, \lambda, 0, \lambda)$*, where* $\lambda, \mu > 0$ *and* $\mu^2 + \lambda^2 = 1$*.*

For $t \to \infty$ *the metrics of this family are approximated arbitrary closely by the direct product* $S^1 \times C(S^2 \times S^3)$*, where* $C(S^2 \times S^3)$ *is the cone over the product of spheres. Moreover, the sphere* S^2 *arises as factorization of the diagonally embedded in* $S^3 \times S^3$ *three-dimensional sphere with respect to the circle action corresponding to the vector field* $\xi^1 + \tilde{\xi}_1$

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Cohomological localization towers

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Homological localizations of spaces or spectra are common tools in Algebraic Topology. If E_* is a (reduced) homology theory, then E_*-localization is an idempotent functor L_E on the homotopy category of spaces or spectra which turns into isomorphisms precisely the E_*-equivalences, i.e., those maps $f\colon X \to Y$ inducing isomorphisms $E_k(X) \cong E_k(Y)$ for all k. Important examples are $E = H\mathbb{Q}$ (a rational Eilenberg–Mac Lane spectrum, for which L_E is ordinary rationalization), and $E = K(n)$, Morava K-theory of height n at some prime.

The existence of *cohomological* localizations, however, has remained unproved since the decade of 1980. It was soon realized that the main difficulties were of set-theoretical nature. In [4] we found that, indeed, Vopěnka's Principle implies that arbitrary cohomological localizations exist. More recently, we proved in [1] that the existence of cohomological localizations follows from a much weaker assumption, namely the existence of a proper class of supercompact cardinals. This is a consequence of the fact that cohomological localizations can be defined by means of a Σ_2-formula in the sense of Lévy complexity. In contrast with this fact, homological localizations are Σ_1-definable and therefore the existence of homological localizations is provable in ZFC, as is well known.

In our talk we will briefly survey these developments and will show how to *construct* a cohomological localization L^E for any spectrum E once its existence is guaranteed. For this, we use a *long tower* of functors that starts with E-completion of each given space X (that is, the total complex of a fibrant replacement of X in the category of cosimplicial spaces with Bousfield's

E-resolution model structure [2]) and goes backwards by means of successive homotopy inverse limits. This method was first used by Dror Farjoun and Dwyer in [5] as an alternative way of obtaining ordinary homological localizations. Assuming the existence of sufficiently large supercompact cardinals, our tower converges for each space X, yielding a cohomological localization $L^E X$ upon stabilization.

This is joint work with Imma Gálvez [3].

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Parallelohedra and the Voronoi Conjecture

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A parallelohedron is a convex polyhedron which tiles the Euclidean space by parallel copies in face-to face way. The concept of parallelohedron was introduced by E. Fedorov, the great Russian crystallographer. He found all the five combinatorial types of 3-dimensional parallelohedra which are of great importance in crystallography. High-dimensional parallelohedra have many applications in geometry of numbers and some other other fields of mathematics. In addition, high-dimensional parallelohedra present an extremely interesting class of polyhedra in itself. An obvious example of a parallelohedron of any dimension is the parallelepiped. Much more interesting type of a parallelohedron is presented by the permutahedron of any dimension. A great contribution to the theory of parallelohedra was made by H. Minkowski, G. Voronoi, B. Delone, A. Alexandrov, O. Zhitomirski, B. Venkov, S. Ryshkov, and others.

At the same time, despite the considerable efforts, the central problem — a proof of the Voronoi conjecture on the affine equivalence of an arbitrary parallelohedron to some Voronoi parallelohedron — remains unresolved. In the talk we are going to present an overview of classical results of the theory of parallelohedra as well as some recent results obtained by A. Garber, A. Magazinov, A. Gavrilyuk, and by the author.

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The Sokolov system and integrable Kirchhoff elasticae via discriminantly separable polynomials and Buchstaber's two-valued groups

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We use the discriminantly separable polynomials of degree two in each of three variables to integrate explicitly the Sokolov case of a rigid body in an ideal fluid and integrable Kirchhoff elasticae in terms of genus two theta-functions. The integration procedure is a natural generalization of one used by Kowalevski in her celebrated 1889 paper. It is based on the characteristic property of the discriminantly separable polynomials, that the discriminants of such polynomials are decomposable as products of polynomials of one variable each. One of the key steps is a change of variables which is an analogue of the Kowalevski magic change of variables. Such a change of variables is an instance of the two-valued group structure associated to an elliptic curve. The two-valued group structures on elliptic curves have been introduced by V. M. Buchstaber.

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On Gromov's macroscopic dimension conjecture

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Gromov's conjecture states that the macroscopic dimension of the universal covering of an n-manifold with positive scalar curvature does not exceed $n - 2$. This conjecture seems out of reach, since it implies the Gromov–Lawson conjecture which is known to be a Novikov type conjecture. We prove Gromov's conjecture for manifolds with some conditions on the fundamental groups. In particular, we prove Gromov's conjecture for duality groups that satisfies the coarse Baum–Connes conjecture.

Characterization of simplicial complexes with Buchstaber number two

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The Buchstaber invariant is a combinatorial invariant of simple polytopes and simplicial complexes that comes from toric topology [1]. With each $(n-1)$-dimensional simplicial complex K on m vertices we can associate a topological space – $(m+n)$-dimensional moment-angle complex $\mathcal{Z}_K$ with a canonical action of a compact torus $T^m = (S^1)^m$. The topology of $\mathcal{Z}_K$ and of the action depends only on the combinatorics of K, which gives a tool to study the combinatorics of polytopes and simplicial complexes in terms of the algebraic topology of moment-angle complexes and vice versa. A *Buchstaber invariant* $s(K)$ is equal to the maximal dimension of torus subgroups $H \subset T^m$, $H \simeq T^k$, that act freely on $\mathcal{Z}_K$. If K is not a simplex, then $1 \leqslant s(K) \leqslant m-n$.

Problem 1 (Victor M. Buchstaber, 2002) *To find an effective description of $s(K)$ in terms of the combinatorics of K.*

We consider the following question: to characterize simplicial complexes K with $s(K) = r$ fixed.

In the important case when K is the boundary of the polytope polar to a simple polytope P we have $s(P) = 1$ if and only if $P = \Delta^n$ or, equivalently, $m - n = 1$. The case $s(P) = 2$ is much more complicated: for any $k \geqslant 2$ there exists an n-polytope P with $m = n + k$ facets and $s(P) = 2$. However any polytope

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with $s(P) = 2$ should satisfy one of the following conditions: either P is a square; or the chromatic number $\gamma(P)$ equals $m-1$ with $m < \frac{3}{2}(n+1)+1$; or any two facets of P intersect, and $m < \frac{7}{4}(n+1)+2$.

In the case of general simplicial complexes situation is non-trivial even for $r = 1$. Let $N(K)$ be the set of all *missing faces* of K, i.e. subsets $\tau \subset \mathrm{vert}(K)$ such that $\tau \notin K$, but $\omega \in K$ for any proper subset $\omega \subset \tau$. The set $N(K)$ uniquely defines K.

Our main result is the characterization of simplicial complexes with $s(K) = 1$ and $s(K) = 2$.

Theorem 1

I. $s(K) = 1$ *iff either* $|N(K)| = 1$*; or* $N(K) = \{\tau_1, \tau_2\}$, $\tau_1 \cap \tau_2 \neq \varnothing$*; or* $|N(K)| \geqslant 3$ *and any three missing faces intersect.*

II. $s(K) = 2$ *iff there exist either two or three missing facets with empty intersection and* $N(K)$ *does not contain any of the following subsets*

1) $\{\tau_1, \tau_2, \tau_3\}$*:* $\tau_1 \cap \tau_2 = \tau_1 \cap \tau_3 = \tau_2 \cap \tau_3 = \varnothing$*;*

2) $\{\tau_1, \tau_2, \tau_3, \tau_4\}$*:* $\tau_1 \cap (\tau_2 \cup \tau_3 \cup \tau_4) = \tau_2 \cap \tau_3 \cap \tau_4 = \varnothing$*;*

3) $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\}$*:* $\tau_1 \cap \tau_2 = \tau_1 \cap \tau_5 = \tau_1 \cap \tau_3 \cap \tau_4 = \tau_2 \cap \tau_3 \cap \tau_5 = \tau_2 \cap \tau_4 \cap \tau_5 = \varnothing$*;*

4) $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6\}$*:* $\tau_1 \cap \tau_3 = \tau_1 \cap \tau_2 \cap \tau_4 = \tau_1 \cap \tau_2 \cap \tau_5 = \tau_1 \cap \tau_4 \cap \tau_6 = \tau_1 \cap \tau_5 \cap \tau_6 = \tau_2 \cap \tau_3 \cap \tau_6 = \tau_3 \cap \tau_4 \cap \tau_5 = \varnothing$*;*

5) $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7\}$*:* $\tau_1 \cap \tau_2 \cap \tau_4 = \tau_1 \cap \tau_3 \cap \tau_5 = \tau_1 \cap \tau_6 \cap \tau_7 = \tau_2 \cap \tau_3 \cap \tau_6 = \tau_2 \cap \tau_5 \cap \tau_7 = \tau_3 \cap \tau_4 \cap \tau_7 = \tau_4 \cap \tau_5 \cap \tau_6 = \varnothing$*.*

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Deformations and Contractions of Algebraic Structures

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Deformations of analytic and algebraic objects is an old problem both in mathematics and physics. In this talk I restrict myself to the case of Lie algebras - one of the most important categories in physics.

Classical 1-parameter deformation theory of associative algebras was worked out by Gerstenhaber back in the 60's, and was soon applied to Lie algebras by Nijenhuis and Richardson. The problem with the classical theory is that it is not satisfactory to describe all nonequivalent deformations of a given object. For that purpose one has to introduce deformations with a complete local algebra base. I am going to define the notion a miniversal deformation, and consider the opposite notion of deformations: contractions, which are the inverse of the so called jump deformations. As from a miniversal deformation one can determine all jump deformations, one can say that miniversal deformations contain all the information about contractions as well. They also carry other interesting information about the moduli space of the objects.

I will demonstrate the notions on some examples, and raise some open questions.

Hopf algebras and homotopy invariants

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This report is based on recent results arising form joint work with Victor Buchstaber.

For an arbitrary topological space X, the loop space homology $H_*(\Omega\Sigma X;\mathbb{Z})$ is a Hopf algebra. We introduce a new homotopy invariant of a topological space X taking for its value the isomorphism class (over the integers) if the Hopf algebra $H_*(\Omega\Sigma X;\mathbb{Z})$. Studying different Hopf algebra structures on a tensor algebra $T(V)$, we analyse homotopy types of topological spaces. In particular, for a given X corresponding invariant is an obstruction to the existence of a homotopy equivalence $\Sigma X \simeq \Sigma^2 Y$ for some space Y. In Toric Topology, we apply our new invariant in characterising the cohomology ring structures of the moment-angle complexes. We further investigate relations between this new homotopy invariant and well known classical invariants and constructions in homotopy theory and algebra.

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Higher order generalized Euler characteristics and generating series

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For a complex quasi-projective manifold with a finite group action, higher order Euler characteristics are generalizations of the orbifold Euler characteristic introduced by physicists. The generating series of the higher order Euler characteristics of a fixed order of the Cartesian products of the manifold with the wreath product actions on them were computed by H. Tamanoi. I'll discuss motivic versions of the higher order Euler characteristics with values in the Grothendieck ring of complex quasi-projective varieties extended by the rational powers of the class of the affine line and give formulae for the generating series of these generalized Euler characteristics for the wreath product actions. The formulae are given in terms of the power structure over the (extended) Grothendieck ring of complex quasi-projective varieties defined earlier.

The talk is based on joint works with I. Luengo and A. Melle-Hernández.

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Kokotsakis polyhedra and elliptic functions

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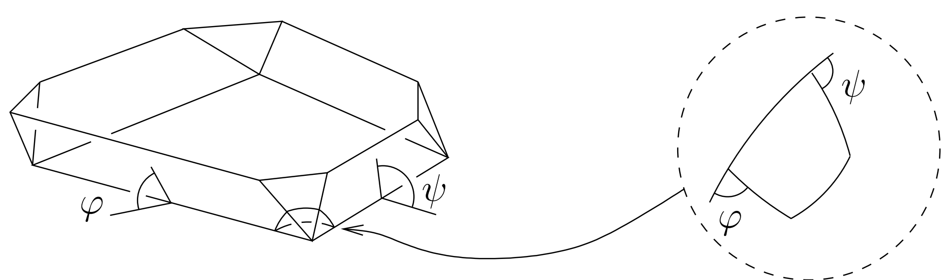
A Kokotsakis polyhedron [1] is a polyhedral surface in $\mathbb{R}^3$, consisting of a quadrilateral and a "belt" of faces surrounding it, so that all interior vertices have degree 4, see Figure. Generically, a Kokotsakis polyhedron is rigid, as can be seen by counting degrees of freedom. Two non-trivial classes of flexible polyhedra were given in [2], but a complete classification was missing.

The list of all flexible Kokotsakis polyhedra was produced in [3]. We describe the main ideas of this work.



Each interior vertex of the polyhedron establishes polynomial relations between the tangents of adjacent dihedral half-angles:

$$c_{22}z^2w^2 + c_{20}z^2 + c_{02}w^2 + 2c_{11}zw + c_{00} = 0, \tag{1}$$

where $z = \tan\frac{\phi}{2}$, $w = \tan\frac{\psi}{2}$, see Figure. Generically, equation (1) describes an elliptic curve with a parametrization

$$\{z = p\,\mathrm{sn}\,t,\ w = q\,\mathrm{sn}(t+t_0)\} \quad \text{or} \quad \{z = p\,\mathrm{cn}\,t,\ w = q\,\mathrm{cn}(t+t_0)\}$$

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Thus with every interior vertex of the polyhedron an elliptic modulus k, two amplitudes p and q, and a phase shift t_0 is associated. This leads to a new class of flexible polyhedra: equal moduli, equal amplitudes at corresponding vertices, and the sum of shifts equal to a period.

One of the flexible cases described by Sauer and Graf involves orthodiagonal quadrilaterals, which have the phase shift equal to a quarter-period. In this case, the moduli are not equal, but flexibility follows from compatibility of involutions on the corresponding elliptic curves.

In [3], a diagram of branched covers between configuration spaces of pieces of a Kokotsakis polyhedron is studied. For example, the configuration space of two adjacent vertices is the fiber product of the configuration spaces of these vertices. Flexibility of a polyhedron means that the diagram is in some sense overdetermined. This provides a clue to the classification.

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Non-compact symplectic toric manifolds

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I will report on joint work with Eugene Lerman in which we extend Delzant's classification of compact symplectic toric manifolds to the non-compact case. The quotient of a symplectic toric manifold by the torus action is a manifold with corners Q. The classification is in terms of a "unimodular local embedding" from Q to the dual of the Lie algebra of the torus, plus a degree two cohomology class on Q. One technical issue is in constructing the smooth structure "upstairs" when Q has infinitely many facets.

Operator pencils on densities

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Let Δ be a linear differential operator acting on the space of densities of a given weight t_0 on a manifold M. One can consider a lifting of this operator to an operator pencil Δ_t such that any operator Δ_t acts on densities of weight t. We study liftings which are equivariant with respect to the group of diffeomorphisms of M and some of its subgroups. Our analysis is essentially based on the simple but very important fact that an operator pencil as above can be identified with a linear differential operator $\hat{\Delta}$ acting on the commutative algebra of densities of all weights, which is endowed with a canonical scalar product.

Equivariant almost complex quasitoric structures

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We investigate the problem of existence of equivariant almost complex structure on a given omnioriented quasitoric manifold [2].

Theorem 1 *The canonical stably complex structure described by Buchstaber, Panov and Ray [1] is equivalent to an almost complex structure if and only if all signs of fixed points are positive.*

We sketch the proof that is itself based on classical obstruction theory reconsidered in equivariant setting. Moreover, the set of all equivariant structures up to homotopy is either empty or infinite and admits affine space structure.

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Homotopy BV algebras in Poisson geometry

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We define and study the degeneration property for $\mathrm{B}V_\infty$ algebras and show that it implies that the underlying L_∞ algebras are homotopy abelian. The proof is based on a generalisation of the well-known identity $\Delta(e^\xi) = e^\xi \left(\Delta(\xi) + \frac{1}{2}[\xi, \xi]\right)$ which holds in all BV algebras. As an application we show that the higher Koszul brackets on the cohomology of a manifold supplied with a generalised Poisson structure all vanish.

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Equivariant cobordism of unitary toric manifolds

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The notion of unitary toric manifolds was introduced by Masuda in [3]. A *unitary toric manifold* of dimension $2n$ is a smooth closed manifold with an effective T^n-action such that its tangent bundle admits a T^n-equivariant stable complex structure. These geometrical objects are the topological analogues of compact non-singular toric varieties, and constitute a much wider class than that of quasi-toric manifolds introduced by Davis and Januszkiewicz in [1]. Also, the nonempty fixed set of a unitary torus manifold must be isolated since the action is assumed to be effective. We shall show that

Theorem 1 *Let M be a unitary torus manifold. Then M bounds equivariantly if and only if the equivariant Chern numbers*

$$\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M] \rangle = 0$$

for all $i, j \in \mathbb{N}$, where $[M]$ is the fundamental class of M with respect to the given orientation.

In addition, we also show that

Theorem 2 *Suppose that M^{2n} is a $(2n)$-dimensional unitary torus manifold. If M does not bound equivariantly, then the number of fixed points is at least $\lceil \frac{n}{2} \rceil + 1$, where $\lceil \frac{n}{2} \rceil$ denotes the minimal integer no less than $\frac{n}{2}$.*

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Remark 1 *This gives a supporting evidence on Kosniowski conjecture, saying that for a unitary S^1-manifold M^n with isolated fixed points, if M^n does not bound equivariantly then the number of fixed points is greater than $f(n)$, where $f(n)$ is some linear function. As was noted by Kosniowski in [2], the most likely function is $f(n) = \frac{n}{4}$, so the number of fixed points of M^n is at least $[\frac{n}{4}] + 1$.*

Let $\mathfrak{U}_{T^n}$ denote the abelian group formed by equivariant unitary cobordism classes of all $2n$-dimensional unitary toric manifolds, and $\mathfrak{M}_{(\mathbb{Z}_2)^n}$ the abelian group formed by equivariant unoriented cobordism classes of all n-dimensional smooth closed manifolds with an effective $(\mathbb{Z}_2)^n$-action. We shall show that there is a natural homomorphism $\Phi : \mathfrak{U}_{T^n} \longrightarrow \mathfrak{M}_{(\mathbb{Z}_2)^n}$. In particular, we show that

Theorem 3 *The homomorphism $\Phi : \mathfrak{U}_{T^n} \longrightarrow \mathfrak{M}_{(\mathbb{Z}_2)^n}$ is onto.*

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Periodic and rapid decay rank two self-adjoint commuting differential operators

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Self-adjoint commuting ordinary differential operators of rank two are considered. We find sufficient conditions when an operator of fourth order commuting with an operator of order 4g + 2 is self-adjoint. An equation on potentials $V(x), W(x)$ of the self-adjoint operator $L_4 = (\partial_x^2 + V(x))^2 + W(x)$ and some additional data is introduced. With the help of this equation operators with polynomial, periodic and rapid decay coefficients are constructed. Some problems related to rank two solutions of soliton equations are discussed.

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Algebraic models of spaces of sections of nilpotent fibrations

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I will describe precise and explicit objects in different algebraic categories describing the rational homotopy type of spaces of sections of nilpotent fibrations. Mapping spaces are particular examples of this class of spaces. The special computability character of these algebraic models let us deduce applications in different settings which will also be presented. This work has been carried out in collaboration with Urtzi Buijs and part of it also with Yves Félix.

Tucker and Fan's lemma for manifolds

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A discrete version of the Borsuk–Ulam theorem is known as *Tucker's lemma*:
Let T be an antipodal triangulation of the d-sphere $\mathbb{S}^d$. Let

$$L : V(T) \to \{+1, -1, +2, -2, \dots, +d, -d\}$$

be an equivariant labelling, i.e. $L(-v) = -L(v)$). Then there exists a "complementary edge" in T, i.e. an edge such that its two vertices are labelled by opposite numbers.

This lemma was extended by Ky Fan in 1952:
Let T be an antipodal triangulation of $\mathbb{S}^d$. Suppose that each vertex v of T is assigned a label $L(v)$ from $\{\pm 1, \pm 2, \dots, \pm n\}$ in such a way that $L(-v) = -L(v)$. If this labelling does not have complementary edges, then there are an odd number of d-simplices of T whose labels are of the form $\{k_0, -k_1, k_2, \dots, (-1)^d k_d\}$, where $1 \le k_0 < k_1 < \dots < k_d \le n$. In particular, $n \ge d + 1$.

Let M be a closed PL-manifold with a free simplicial involution $T : M \to M$, i.e. $T^2(x) = x$ and $T(x) \neq x$ for all $x \in M$. For any $\mathbb{Z}_2$-manifold (M, T) we say that a map $f : M^d \to \mathbb{R}^n$ is *antipodal* (or equivariant) if $f(T(x)) = -f(x)$.

We say that a closed PL free $\mathbb{Z}_2$-manifold (M^d, T) is a *BUT (Borsuk-Ulam Type) manifold* if for any continuous $g : M^d \to \mathbb{R}^d$ there is a point $x \in M$ such that $g(T(x)) = g(x)$. Equivalently,

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if a continuous map $f : M^d \to \mathbb{R}^d$ is antipodal, then the zeros set $Z_f := f^{-1}(0)$ is not empty.

In [1], we found several equivalent necessary and sufficient conditions for manifolds to be BUT. For instance, *M^d is a BUT manifold if and only if M admits an antipodal continuous transversal to zeros map $h : M^d \to \mathbb{R}^d$ with $|Z_h| = 2 \pmod 4$.*

One of extensions of Tucker and Fan's lemma for manifolds is the following:

Theorem 1 *Let M^d be a closed PL BUT manifold with a free involution. Let Λ be any equivariant triangulation of M. Let $L : V(\Lambda) \to \{+1, -1, +2, -2, \ldots, +n, -n\}$ be an equivariant labelling. Then there is a complementary edge or an odd number of d-simplices whose labels are of the form $\{k_0, -k_1, k_2, \ldots, (-1)^d k_d\}$, where $1 \le k_0 < k_1 < \ldots < k_d \le n$.*

Now we extend this theorem for the case $d > n$. We say that a closed PL-free $\mathbb{Z}_2$-manifold (M^d, T) is a $\mathrm{BUT}_{d,n}$ if for any continuous $g : M^d \to \mathbb{R}^n$ there is a point $x \in M$ such that $g(T(x)) = g(x)$.

In [1] (in terms of the equivariant cobordism theory) we found a sufficient condition for (M^d, T) to be $\mathrm{BUT}_{d,n}$.

Theorem 2 *Let $n < d$. Let Λ be any equivariant triangulation of a $BUT_{d,n}$ manifold (M^d, T). Let*

$$L : V(\Lambda) \to \{+1, -1, +2, -2, \ldots, +n, -n\}$$

be any equivariant labelling of Λ. Then there exists a complementary edge in Λ.

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A refined Riemann's singularity theorem for sigma function

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We study sigma functions with arbitrary characteristics of a compact Riemann surface from the view point of the tau and Schur functions.

Let X be a compact Riemann surface of genus $g \geq 1$, p_∞ a point of X, $w_1 < \cdots < w_g$ the gap sequence at p_∞, $\{\alpha_i, \beta_i\}$ a canonical homology basis, Δ the Riemann divisor, $L = q_1 + \cdots + q_{g-1} - (g-1)p_\infty$ a flat line bundle of degree 0 and $0 \leq b_1 < \cdots < b_g$ the gap sequence of $H^0(X, L(*p_\infty))$ and $m = \dim H^0(X, \mathcal{O}(q_1 + \cdots + q_{g-1}))$.

We study properties of the Schur function $s_\lambda(t)$ associated with the partition
$\lambda = (b_g, ..., b_1) - (g-1, ..., 1, 0)$.

We show the followings.

(i) $s_\lambda(t)$ is a polynomial of t_{w_1},...,t_{w_g}.

(ii) For any partition μ such that $\lambda \leq \mu$, any integer $n < m$ and any set $\{i_1, ..., i_n\}$ of cardinality n with $i_j \in \{w_k\}$,

$$\partial_{i_1} \cdots \partial_{i_n} s_\mu(0) = 0,$$

where $\partial_i = \partial/\partial t_i$.

(iii) There exists an explicitly constructed sequence $a_1 > \cdots > a_g$ with $a_i \in \{w_j\}$ such that $\partial_{a_1} \cdots \partial_{a_m} s_\lambda(0) = \pm 1$.

By making use of the properties of the tau function of the KP-hierarchy we can deduce, from the above properties of Schur

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functions, the following results for sigma functions of X. Let

$$e = \sum_{i=1}^{g-1} q_i - \Delta.$$

(i)' The first term of the expansion of the sigma function with the characteristics e is $s_\lambda(u)$, $u = (u_{w_1}, ..., u_{w_g})$.

(ii)' For any $n < m$ derivatives of $\sigma(u)$ of degree n at e all vanish.

(iii)' $\partial_{u_{a_1}} \cdots \partial_{u_{a_m}} \sigma(e) = \pm 1$, where $a_1, , , a_m$ are the same as in (iii).

The properties (ii)' and (iii)' imply the Riemann's vanishing theorem for the theta function. In particular (iii)' gives not only the existence of the non-vanishing derivative of degree m but also gives it explicitly.

Geometric differential equations on the universal spaces of Jacobians of elliptic and hyperelliptic curves

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In [1] a method of construction of the Gauss-Manin connection on the universal spaces of Jacobians of (n, s)-curves based on the theory of sigma-functions of such curves is given. The problem of constructing a metric $G = (g_{i,j})$ in the space of parameters that agrees with the Gauss-Manin connection on the universal bundle of Jacobians naturally arises. In the talk we give the solition of this problem in the case of curves of genus 1 and 2. We derive and study the differential equations defining this metric. A description of solutions of the differential equations obtained will be given.

The genus one case will result in the equation

$$8(z-1)z^2 f''' + 4z(9z-4)f'' + \left(26z - \frac{16}{9}\right) f' + f = 0. \quad (1)$$

The solution of this equation is the coefficient $g_{1,2}$ of the metric G in coordinates related to vector fields on the basis tangent to the discriminant of the elliptic curve

$$y^2 = 4x^3 - g_2 x - g_3.$$

The general solution of equation (1) is described in terms of generalized hypergeometric series ${}_3F_2(a, a, a; b, c; z)$ for constant a, b, c.

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Another choice of coordinates gives on $g_{1,2} = f(z)$ the equation

$$9g_3 f''' - 3g_2^2 f'' + 3g_3 g_2 f' + (9g_3^2 - g_2^3) f = 0, \qquad (2)$$

where $g_2 = g_2(z) = 3\wp(z+c_0; 0, c_3)$, $g_3 = g_3(z) = \frac{1}{2}\wp'(z+c_0; 0, c_3)$, and c_0, c_3 are constant.

Using the methods described in [2], we will show how in the frame of our consideration of the heat equation the Chazy-3 equation arises from vector fields tangent to the discriminant of the elliptic curve.

In the case of genus 2 for the operator $l_2 =$

$$6\lambda_6 \partial_{\lambda_4} + \left(8\lambda_8 - \frac{12}{5}\lambda_4^2\right)\partial_{\lambda_6} + \left(10\lambda_{10} - \frac{8}{5}\lambda_4\lambda_6\right)\partial_{\lambda_8} - \frac{4}{5}\lambda_4\lambda_8\partial_{\lambda_{10}},$$

which is tangent to the discriminant of the curve

$$y^2 = x^5 + \lambda_4 x^3 + \lambda_6 x^2 + \lambda_8 x + \lambda_{10},$$

we obtain a system of equations on the matrix G that is analogous to equation (2). This system is linear with coefficiens polynomial in λ_4 and λ_8, moreover, λ_4 is defined as a solution of equation

$$5f'''' + 248 f f'' + 208(f')^2 + 576 f^3 = 0$$

and $\lambda_8 = \frac{1}{48}\lambda_4'' + \frac{3}{10}\lambda_4^2$.

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Non-symplectic smooth group actions on symplectic manifolds

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We construct smooth actions of any compact Lie group G on symplectic manifolds, including complex projective spaces, such that the actions are not symplectic with respect to any possible symplectic structure on the manifolds in question. Such examples were obtained for the first time in [1] in the case where $G = S^1$, and then in [2] for any compact Lie group G.

More precisely, we proof the following two theorems.

Theorem 1 *Let $G = S^1$. Then there exists a smooth action of G on a product $S^2 \times \cdots \times S^2$ of the 2-sphere $S^2 = \mathbb{C}P^1$ such that the action is not symplectic with respect to any possible symplectic structure on $S^2 \times \cdots \times S^2$.*

Theorem 2 *Let G be a compact Lie group. Then there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the action is not symplectic with respect to any possible symplectic structure on $\mathbb{C}P^n$.*

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Toric geometry of the action of maximal compact torus on complex Grassmannians

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It is very known and classical problem to describe the topological structure of the orbit space of the Grassmannian $G_{n,k}$ by the canonical action of compact torus T^n. We consider this problem and approach it from the point of view of toric geometry and toric topology (see [1]) using techniques based on the notion of moment map. Since $G_{n,1} = \mathbb{C}P^{n-1}$ is a toric manifold whose topological structure is known and easy to describe, we assume that $k \geq 2$.

In the first part of the talk we present methods and results of algebraic geometry we use. More closely, the action of a compact torus T^n on $G_{n,k}$ extends to the action of an algebraic torus $(\mathbb{C}^*)^n$ for which the closures of the orbits are toric varieties. This algebraic torus action is also known to produce the equivariant partition of the Grassmann manifolds into the strata. On the other hand there is the moment map (see [2]) $\mu : G_{n,k} \to P^{n-1} \subset \mathbb{R}^{n-1}$, where P^{n-1} is a convex polytope which is the image of the so-called main stratum. The image of the closure of any algebraic torus orbit, as well as of the stratum which contains it, is a polytope contained in P^{n-1}, but this polytope in general does not have to be a face of P^{n-1}. The combinatorics of the polytope P^{n-1} does not determine the lattice of the strata on $G_{n,k}$ as it is in the case of toric manifolds, and therefore we distinguish the class of admissible polytopes in P^{n-1} which are in the image of the moment map. The moment map is equivariant under the action of compact torus giving the homeomorphism between T^n-orbit spaces of the closures of algebraic torus orbits and admissible polytopes from P^{n-1}.

In the second part of the talk we demonstrate the application of these methods in case of the orbit space for the first non-trivial case $(n,k) = (4,2)$ and show that it leads to important and surprising results. More precisely, we prove that $G_{4,2}/T^4$ is homeomorphic to the $\mathbb{C}P^1$ - family of octahedra P^3 glued to each other along the whole boundary and that this gluing produce the cell decomposition of the topological manifold S^5. From the general results of algebraic topology it is known that any manifold homeomorphic to sphere S^5 has unique smooth structure. But, the canonical projection from $G_{4,2}$ to this sphere S^5 is not a smooth map. In that context our approach together with the known results on local structure of the orbit space for compact group actions, enables us to explicitly describe the differentiable structure of $G_{4,2}/T^4$ coming from the smooth structure of $G_{4,2}$.

The talk is based on the joint work with Victor M. Buchstaber.

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Quotients of affine spherical varieties by unipotent subgroups

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A stratification of a topological space or an algebraic variety X is in general a decomposition of X into a (locally) finite disjoint union of locally closed subsets which have a simple structure. Stratifications are of great use in algebraic topology and geometry. If a connected solvable complex algebraic group B acts on a complex algebraic variety X with finitely many orbits, then the orbital decomposition provides a nice stratification of X with strata isomorphic to products of several copies of $\mathbb{C}$ and $\mathbb{C} \setminus \{0\}$. Homological and intersection-theoretic properties of X are quite nice in this case.

Usually the finiteness property is not easy to check, but certain assumptions guarantee it. Namely suppose that X is a *spherical* variety, i.e., a connected reductive algebraic group G acts on X in such a way that a Borel subgroup $B \subset G$ has a dense open orbit in X. A theorem of Brion and Vinberg says that B acts on X with finitely many orbits.

We are interested in pushing down the finiteness property to quotient varieties. Let X be an *affine* algebraic variety acted on by an algebraic group H. If the algebra of invariant polynomials $\mathbb{C}[X]^H$ is finitely generated, then one may consider its spectrum $X/\!\!/H = \operatorname{Spec} \mathbb{C}[X]^H$, called the *categorical quotient* of X by H. Morally, the categorical quotient should be thought of as the space of orbits for the action, but in practice, the situation is more delicate. In particular, the natural quotient map $\pi : X \to X/\!\!/H$ is not always surjective.

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Now suppose that X is an affine spherical G-variety and $H \subset B$ is a normal unipotent subgroup. Then $X/\!\!/H$ is well defined and B/H acts on $X/\!\!/H$ in a natural way. It is natural to ask whether this action has finitely many orbits, as B had on X. This question was raised recently by Panyushev. He conjectured that the answer is affirmative for $H = [U, U]$, the commutator of the maximal unipotent subgroup $U \subset B$. We disprove this conjecture. Our first main result is:

Theorem 1 *For a fixed reductive group G, the categorical quotient $X/\!\!/[U, U]$ of any affine spherical G-variety X has finitely many $B/[U, U]$-orbits if and only if all simple factors of G are of types $\mathsf{A}_1, \mathsf{B}_2, \mathsf{G}_2$.*

Since Theorem 1 is somewhat disappointing, because we would like to have an affirmative answer to Panyushev's question, it is natural to ask whether it can be obtained under some restrictions on X. The restrictions which we impose are of representation-theoretic nature. Recall from the representation theory of reductive groups that the irreducible representations of G are parameterized by their *highest weights*, which are the dominant weights of the root data of G. For any dominant weight λ, we consider the subdiagram of the Dynkin diagram of G whose nodes correspond to the fundamental weights occurring in the decomposition of λ with positive coefficients, and call it the *support* of λ.

Theorem 2 *Suppose that the supports of all highest weights of the irreducible G-modules occurring in $\mathbb{C}[X]$ do not contain subdiagrams of type A_2. Then $B/[U, U]$ acts on $X/\!\!/[U, U]$ with finitely many orbits.*

This theorem can be extended to quotients by intermediate subgroups $U \supseteq H \supseteq [U, U]$. It applies, for instance, to $X = \mathrm{GL}_{2n}(\mathbb{C})/\mathrm{Sp}_{2n}(\mathbb{C})$, the moduli space of symplectic structures on $\mathbb{C}^{2n}$.

Brunnian and Conen braids and homotopy groups of spheres

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Let M be a connected surface, possibly with boundary, and let $B_n(M)$ denote the n-strand braid group on a surface M. A *Brunnian braid* means a braid that becomes trivial after removing any one of its strands. Let $\mathrm{Brun}_n(M)$ denote the set of the n-strand Brunnian braids. Then $\mathrm{Brun}_n(M)$ forms a subgroup of $B_n(M)$. Let $A_{i,j}$ for $1 \leq i < j \leq n$ be the set of standard generators of the pure braid group of the disc. We define $A_{i,j}[M] = f_*(A_{i,j})$ where f is an inclusion $f\colon D^2 \hookrightarrow M$ of a disc into the surface M. Let $\langle\langle A_{i,j}[M]\rangle\rangle^P$ be the normal closure of $A_{i,j}[M]$ in $P_n(M)$, the pure braid group of the surface M.

Given a group G, and a set of normal subgroups $H_1, \ldots, H_n$, ($n \geq 2$), the symmetric commutator product of these subgroups is defined as

$$[H_1, \ldots, H_n]_S := \prod_{\sigma \in \Sigma_n} [[H_{\sigma(1)}, H_{\sigma(2)}], \ldots, H_{\sigma(n)}],$$

where Σ_n is the symmetric group of degree n.

Theorem 1 *For $n \geq 2$ let*

$$R_n(M) = [\langle\langle A_{1,n}[M]\rangle\rangle^P, \langle\langle A_{2,n}[M]\rangle\rangle^P, \ldots, \langle\langle A_{n-1,n}[M]\rangle\rangle^P]_S$$

1. If $M \neq S^2$ or $\mathbb{R}\mathrm{P}^2$, then $\mathrm{Brun}_n(M) = R_n(M)$.

2. If $M = S^2$ and $n \geq 5$, then there is a short exact sequence

$$R_n(S^2) \hookrightarrow \mathrm{Brun}_n(S^2) \twoheadrightarrow \pi_{n-1}(S^2).$$

3. If $M = \mathbb{R}\mathrm{P}^2$ and $n \geq 4$ then there is a short exact sequence

$$R_n(\mathbb{R}\mathrm{P}^2) \hookrightarrow \mathrm{Brun}_n(\mathbb{R}\mathrm{P}^2) \twoheadrightarrow \pi_{n-1}(S^2).$$

Let $\alpha \in B_{n-1}(M)$ be an arbitrary braid. We are looking for a solution of the following system of equations

$$\begin{cases} d_1(\beta) = \alpha, \\ \dots \\ d_n(\beta) = \alpha, \end{cases} \tag{1}$$

where the operations $d_i\colon B_n(M) \to B_{n-1}(M)$ are obtained by forgetting the i-th strand, $1 \leq i \leq n$.

Theorem 2 *Let M be any connected 2-manifold such that $M \neq S^2$ or $\mathbb{R}\mathrm{P}^2$ and let $\alpha \in B_{n-1}(M)$. Then the equation (1) for n-strand braids β has a solution if and only if α satisfies the condition that*

$$d_1\alpha = \ldots = d_{n-1}\alpha.$$

Define a set

$$\mathfrak{H}_n^B(M) = \{\beta \in B_n(M) \mid d_1\beta = d_2\beta = \cdots = d_n\beta\}.$$

Namely $\mathfrak{H}_n^B(M)$ consists of n-strand pure braids such that it stays the same braid after removing any one of its strands, we call this *Cohen braids*. We also study generating sets for Cohen braids.

The talk is based on the joint works with V. G. Bardakov, R. Mikhailov and Jie Wu, in particular, [1].

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On hyperbolic 3-manifolds with geodesic boundary

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The investigation of hyperbolic 3-manifolds with geodesic boundary was started in Thurston's Lecture notes on geometry and topology of 3-manifolds. The computer-based tabulation of them was done by Frigerio, Martelli, and Petronio in [1]. An infinite family was constructed by Paoluzzi and Zimmermann in [2]. The boundary surfaces of manifolds from [2] are knotted similar to Suzuki's spatial θ_n-graphs.

We will discuss two most useful invariants of hyperbolic 3-manifolds – volume and complexity. In the case of closed hyperbolic 3-manifolds volume formulas admit to get linear estimations of complexity [3].

In the present talk we discuss these invariants for Paoluzzi–Zimmermann manifolds and their generalizations. We obtain volume formulas and complexity for two infinite families of hyperbolic 3-manifolds with geodesic boundary [4].

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Algebra of Densities

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The talk is devoted to natural differential-geometric constructions on the algebra of densities, which is a commutative algebra canonically associated with a given manifold or supermanifold.

A density of weight λ is a geometric object which in local coordinates has the form $f(x)|Dx|^{\lambda}$. Here λ is an arbitrary real number. It is clear that densities can be multiplied so that their weights are added. The resulting commutative *algebra of densities* $\mathfrak{F}(M)$, graded by real numbers, possesses a natural invariant scalar product. This algebra is of course a subalgebra of the algebra of functions on a manifold $\hat{M}$, the total space of the frame bundle for the bundle of volume forms on a manifold M. (The manifold $\hat{M}$ is sometimes referred to as the "Thomas bundle" of M.) The existence of the scalar product on the algebra $\mathfrak{F}(M)$ leads to important geometric consequences. Namely, for the Lie algebra of vector fields on $\hat{M}$ – or the derivations of $\mathfrak{F}(M)$ – there arises a canonical divergence operator and for the corresponding algebra of multivector fields, a canonical odd Laplacian (a Batalin-Vilkovisky type operator). From there we can extract classification theorems. For weight $\lambda \neq 1$, the divergence-free derivations of $\mathfrak{F}(M)$ are in a one-to-one correspondence with the vector densities of weight λ on M. A similar statement holds for multivector fields on $\hat{M}$ and multivector densities on M. In particular, the commutator of vector fields on M and the canonical Schouten bracket naturally extend to brackets of vector or multivector densities, respectively. One application of that is the possibility of lifting an even Poisson structure on M to the algebra of densities $\mathfrak{F}(M)$.

These constructions for the algebra of densities have a similarity with the Nijenhuis theorem on derivations of the algebra of forms and the construction of the Nijenhuis bracket. We can see it as two particular instances of "second-order geometry", i.e., geometry arising from iteration of natural first-order constructions for a manifold M (such as taking TM or T^*M, for example). (The idea of second-order geometry has been stressed by K. C. H. Mackenzie.)

(The talk is based on results due to the speaker and H.M.Khudaverdian, and a recent result of our joint student A. Biggs.)

Combinatorial group theory and the homotopy groups of finite complexes

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For $n > k \geq 3$, we construct a finitely generated group with explicit generators and relations obtained from braid groups, whose center is exactly $\pi_n(S^k)$. Our methods can be extended for obtaining combinatorial descriptions of homotopy groups of finite complexes. As an example, we also give a combinatorial description of the homotopy groups of Moore spaces.

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Flat polyhedral complexes

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A *flat polyhedral complex* E arises if several copies of a convex polyhedron (convex body) B are glued together along some of their common faces (closed convex subsets of their boundaries). By construction there is a 'folding map' $p : E \to B$ which resembles the moment map from toric geometry. Examples of flat polyhedral complexes include 'small covers' and other locally standard $\mathbb{Z}_2$-toric manifolds, but the idea can be also traced back (at least) to A. D. Alexandrov's 'flattened convex surfaces'.

A class of flat polyhedral complexes, modelled on a product of two (or more) simplices $B = \Delta^p \times \Delta^q$, was used in [2] for a proof of a multidimensional generalization of Alon's 'splitting necklace theorem' [1].

We show how flat polyhedral complexes and their relatives can be used as 'configuration spaces', leading to new 'fair division theorems', the simplest example being a 'polyhedral curtain division' for two players (Theorem 1).

Definition 1 *Let* $\Delta = \mathrm{conv}\{a_0, a_1, \ldots, a_d\} \subset \mathbb{R}^d$ *be a non-degenerate simplex with the barycenter at the origin. For each pair* $\theta = (F_1, F_2)$ *of complementary faces of* Δ *there is a join decomposition* $\Delta = F_1 * F_2$. *Assuming that both* F_1 *and* F_2 *are non-empty let* $S_\theta^{d-2} = \partial(F_1) * \partial(F_2) \subset \partial(\Delta)$ *be an associated* $(d-2)$*-dimensional, polyhedral sphere. A polyhedral hypersurface* $H \subset \mathbb{R}^d$ *is called a* Δ-curtain *if for some* $\theta = (F_1, F_2)$ *and* $x \in \mathbb{R}^d$

$$H = x + C_\theta = x + \mathrm{cone}(S_\theta^{d-2}). \tag{1}$$

Theorem 1 *Suppose that $\Delta \subset \mathbb{R}^d$ is a simplex with the barycenter at the origin. Let $\mu_1, \mu_2, \ldots, \mu_d$ be a collection of continuous mass distributions (measures) on R^d. Then there exists a Δ-curtain $H = H_{(x,\theta)}$ which divides the space $\mathbb{R}^d$ into two 'half-spaces' H^+ and H^- such that for each $j \in \{1, \ldots, d\}$,*

$$\mu_j(H^+) = \mu_j(H^-).$$

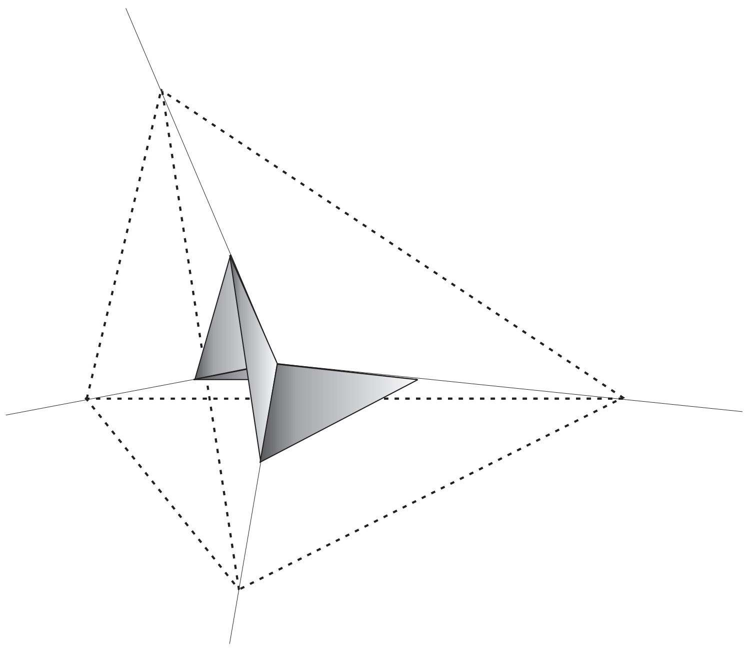

Figure 1: A fragment of a Δ-curtain in $\mathbb{R}^3$.

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Posters

Stanley–Reisner rings of spherical nerve-complexes

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There exists a construction which associates a simplicial complex K_P to each convex polytope P. For a simple polytope P the complex K_P coincides with the boundary ∂P^* of a polar dual polytope. In this case K_P is a simplicial sphere and its Stanley–Reisner ring $\mathbf{k}[K_P]$ is Cohen–Macaulay. The global problem is to describe the properties of a simplicial complex K_P and its Stanley–Reisner ring for general convex polytope P. We develop a simple method to find the depth of $\mathbf{k}[K]$ for any simplicial complex K. This method implies an equality $\operatorname{depth}\mathbf{k}[K_P] = \dim P$ for each convex polytope P.

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Homogeneous extensions of the first order ODE's

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Properties of the first order differential equations

$$y' = \frac{b_0 + b_1\,x + b_2\,y(x) + b_{11}\,x^2 + b_{12}\,xy(x) + b_{22}\,y(x)^2}{a_0 + a_1\,x + a_2\,y(x) + a_{11}\,x^2 + a_{12}\,xy(x) + a_{22}\,y(x)^2} \quad (1)$$

which are equivalent to the systems of the first order equations

$$\dot{y} = 4Z(t)\left(b_0 + b_1 x + b_2 y + b_{11} x^2 + b_{12} xy + b_{22} y^2\right),$$

$$\dot{x} = 4Z(t)\left(a_0 + a_1 x + a_2 y + a_{11} x^2 + a_{12} xy + a_{22} y^2\right), \quad (2)$$

where $Z(t)$-is arbitrary and $x = x(t), y = y(t)$ are considered. After introducing of the projective coordinates $X(t) = x(t)Z(t)$, $Y(t) = y(t)Z(t)$ the system (2) can be presented in form of homogeneous system of equations

$$\dot{X} = 4\,a_0\,Z^2 + (4\,a_2\,Y + (3\,a_1 - b_2)\,X)\,Z + 4\,a_{22}\,Y^2 +$$

$$+ (3\,a_{12} - 2\,b_{22})\,XY + (2\,a_{11} - b_{12})\,X^2 = P(X, Y, Z),$$

$$\dot{Y} = 4\,b_0\,Z^2 + (4\,b_1\,X + (3\,b_2 - a_1)\,Y)\,Z + 4\,b_{11}\,X^2 +$$

$$+ (3\,b_{12} - 2\,a_{11})\,XY + (2\,b_{22} - a_{12})\,Y^2 = Q(X, Y, Z),$$

$$\dot{Z} = -\,(b_2 + a_1)\,Z^2 - (2\,b_{22} + a_{12})\,YZ -$$

$$-(2\,a_{11} + b_{12})\,XZ = R(X, Y, Z) \quad (3)$$

with the condition on the right parts $\partial_X P + \partial_Y Q + \partial_Z R = 0$. As it was shown in [1] properties of the system (2) depend on solutions of the nonlinear partial differential equation

$$P(X, Y, Z)Z_X + Q(X, Y, Z)Z_Y - R(X, Y, Z) = 0. \quad (4)$$

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Theorem 1 *Solutions of the equation (4) are expressed through the solutions of algebraic the first order ODE*

$$\Big(\big(a_{22}\,t^2+a_0+a_2\,t\big)\,A\,(t)+b_{22}\,t^2+t b_2+b_0\Big)\,\dot{A}\,(t)^2+$$

$$+\left((-2\,a_{22}\,t-a_2)\,A\,(t)^2+((a_{12}-2\,b_{22})\,t-b_2+a_1)\,A\,(t)\right)\dot{A}(t)+$$

$$+\,(b_1+b_{12}\,t)\,\dot{A}\,(t)+$$

$$+a_{22}\,A\,(t)^3+(-a_{12}+b_{22})\,A\,(t)^2+(a_{11}-b_{12})\,A\,(t)+b_{11}=0 \quad (5)$$

with respect to the function $A(t)$. With a help of solutions $A(t)$ parametric presentation of the function $Z(X,Y)$ is constructed.

In general case the algebraic curves $H(\dot{A},A,t)=0$ which correspond to the equation (4) are elliptic curves at every value of variable t. They have genus $g=1$ and all their properties depend on the values of the coefficients a_i, a_{ij} and b_i, b_{ij} of the equation (1). So to the cubic ODE's corresponding algebraic curve in general has genus $g=3$. Consideration of homogeneous extensions of $3D$-polynomial systems like the Lorenz and the Rössler systems of equations also is possible. Their studying on the method of the Kowalevskaya exponents is founded .

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Mixed Hodge structures on complements of coordinate complex subspace arrangements

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Let $\mathcal{K}$ be an arbitrary simplicial complex on the set $[n] = \{1, \ldots, n\}$. Define a coordinate planes arrangement

$$Z_{\mathcal{K}} := \bigcup_{\sigma \notin \mathcal{K}} L_{\sigma},$$

where $\sigma = \{i_1, \ldots, i_m\} \subseteq [n]$ is a subset in $[n]$ such that σ does not define a simplex in $\mathcal{K}$ and

$$L_{\sigma} = \{z \in \mathbb{C}^n : z_{i_1} = \cdots = z_{i_m} = 0\}.$$

Any arrangement of complex coordinate subspaces in $\mathbb{C}^n$ can be defined in this way.

In [1] V.M. Buchstaber and T.E. Panov computed cohomology of $\mathbb{C}^n \setminus Z_{\mathcal{K}}$ in terms of cohomology of cocell complex of moment-angle complex $\mathcal{Z}_{\mathcal{K}}$. Also they introduced a bigrading on cohomology of $\mathbb{C}^n \setminus Z_{\mathcal{K}}$. This bigrading was obtained originally from the combinatorial and topological ideas.

Our main result is a computation of the mixed Hodge structure on $H^*(\mathbb{C}^n \setminus Z_{\mathcal{K}})$, we prove that the mixed Hodge structure on $H^*(\mathbb{C}^n \setminus Z_{\mathcal{K}})$ is described completely in terms of the bigrading that was mentioned above.

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Subword complexes and 2-truncations

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Subword complexes were introduced in [4] in the context of matrix Schubert varieties. They reflect interesting combinatorial properties of Coxeter groups. A subword complex $\Delta(\mathbf{Q}, \pi)$ is associated with a pair $(\mathbf{Q}, \pi)$, where $\mathbf{Q}$ is a word in the alphabet of simple reflections and π is an element of a Coxeter group W. The simplices in $\Delta(\mathbf{Q}, \pi)$ correspond to the subwords in $\mathbf{Q}$ whose complements contain reduced expressions of π.

Theorem 1 *Assume that words $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are related by a braid move. Then, under certain conditions, $\Delta(\mathbf{Q}_2, \pi)$ can be obtained from $\Delta(\mathbf{Q}_1, \pi)$ by a composition of edge subdivisions and inverse edge subdivisions.*

Theorem 1 holds for all braid moves in the simply-laced case. In terms of simple polytopes, an edge subdivision is dual to a truncation of a face of codimension 2, or simply a 2−truncation.

Corollary 1 *If $\Delta(\mathbf{Q}_1, \pi)$ and $\Delta(\mathbf{Q}_2, \pi)$ are dual to simple polytopes P_1 and P_2, then there is a polytope $\widetilde{P}$ which can be obtained from each of P_1, P_2 by a sequence of 2−truncations.*

We have a simple explicit description of these sequences of subdivisions and truncations. We apply Corollary 1 to complexes of the form $\Delta(\mathbf{c}\,\mathbf{w}_o; w_o)$, where $\mathbf{c}$ and $\mathbf{w}_o$ are reduced expressions of a Coxeter element and of the longest element w_o, respectively.

Theorem 2 *Any complex of the form $\Delta(\mathbf{c}\,\mathbf{w}_o; w_o)$ is polar dual to a 2−truncated cube.*

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2−truncated cubes form an important class of flag simple polytopes containing such families as flag nestohedra and graph-cubeahedra. Its various properties are discussed in [1].

Generalized associahedra are central objects in cluster combinatorics. They are defined for any finite Coxeter group. They are dual to complexes $\Delta(\mathbf{c}\,\mathbf{w}_o; w_o)$, for certain choice of the expressions $\mathbf{c}, \mathbf{w}_o$.

Corollary 2 *For any Coxeter group* W, *the generalized associahedron of type* W *is a* 2−*truncated cube.*

Partial cases of Corollary 2 were proved in [2] and [5]. The talk is based on the preprint [3] and on some work in progress.

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The equivariant cohomology rings of $(n-k,k)$ Springer varieties

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Given a nilpotent operator $N\colon \mathbb{C}^n \to \mathbb{C}^n$, the Springer variety $\mathcal{S}_N$ is defined to be the following subvariety of a flag variety $Flags(\mathbb{C}^n)$:

$$\mathcal{S}_N = \{V_\bullet \in Flags(\mathbb{C}^n) \mid NV_i \subseteq V_i \text{ for all } 1 \leq i \leq n\},$$

where $V_\bullet = (V_i) : 0 \subseteq V_1 \subseteq V_2 \subseteq \cdots \subseteq V_{n-1} \subseteq V_n = \mathbb{C}^n$ such that $\dim_{\mathbb{C}} V_i = i$. When N is a nilpotent matrix in a Jordan canonical form with weakly decreasing sizes $(\lambda_1, \ldots, \lambda_q)$ of Jordan blocks, $\mathcal{S}_N$ is called the $(\lambda_1, \ldots, \lambda_q)$ Springer variety. It admits an S^1-action induced from the natural T^n-action on $Flags(\mathbb{C}^n)$, where the S^1-subgroup of T^n is given by

$$\{(g^n, g^{n-1}, \ldots, g) \mid g \in \mathbb{C},\ |g| = 1\}.$$

Our concern is an explicit description of the S^1-equivariant cohomology ring of $\mathcal{S}_N$.

Theorem 1 *Let $\mathcal{S}_N$ be the $(n-k,k)$ Springer variety with $0 \leq k \leq n/2$. Suppose that the projection map $H^*_{T^n}(Flags(\mathbb{C}^n);\mathbb{C}) \to H^*_{S^1}(\mathcal{S}_N;\mathbb{C})$ induced from the inclusions of groups $S^1 \to T^n$ and spaces $\mathcal{S}_N \to Flags(\mathbb{C}^n)$ is surjective. Then*

$$H^*_{S^1}(\mathcal{S}_N;\mathbb{C}) \cong \mathbb{C}[\tau_1, \ldots, \tau_n, t]/I,$$

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where $H^*_{S^1}(pt;\mathbb{C}) = \mathbb{C}[t]$ *and* I *is an ideal of the polynomial ring* $\mathbb{C}[\tau_1,\dots,\tau_n,t]$ *generated by the following three types of elements:*

$$\sum_{1\le i\le n}\tau_i - n(n+1)t/2,$$
$$(\tau_i+\tau_{i+1}-(n+k+1-i)t)(\tau_i-\tau_{i+1}-t) \quad (1\le i\le n),$$
$$\prod_{1\le j\le k+1}(\tau_{i_j}-(n+j-i_j)t) \quad (1\le i_1<\cdots<i_{k+1}\le n),$$

where $\tau_{n+1}=0$.

The assumption in the theorem above is known to be satisfied when $k=1,2$ ([1]) but unknown for other values of k. A main tool to prove the theorem is the following commutative diagram:

$$\begin{array}{ccc} H^*_{T^n}(Flags(\mathbb{C}^n);\mathbb{C}) & \longrightarrow & H^*_{T^n}(Flags(\mathbb{C}^n)^{T^n};\mathbb{C}) = \bigoplus_{w\in S_n}\mathbb{C}[t_1,\dots,t_n] \\ \downarrow & & \downarrow \\ H^*_{S^1}(\mathcal{S}_N;\mathbb{C}) & \longrightarrow & H^*_{S^1}(\mathcal{S}_N^{S^1};\mathbb{C}) = \bigoplus_{w\in\mathcal{S}_N^{S^1}}\mathbb{C}[t] \end{array}$$

where all the maps are induced by inclusion maps. Here, the horizontal maps are injective since odd degree cohomology groups of $Flags(\mathbb{C}^n)$ and $\mathcal{S}_N$ vanish, and the right vertical map can be described explicitly, so one can find the image of the bottom horizontal map if the left vertical map is surjective.

This is a joint work with Yukiko Fukukawa.

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Combinatorial commutative algebra of some moment-angle manifolds and their topological types

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A moment-angle manifold $\mathcal{Z}_P$ corresponding to a simple polytope P is one of the main objects of study in *Toric Topology* [2]. The topology of $\mathcal{Z}_P$ and the combinatorial structure of the polytope P are closely connected with each other by means of commutative algebra of the face ring $k[K_P]$.
A generalized truncation polytope is a k-vertex cut of a product of two simplices $P = vc^k(\Delta^{n_1} \times \Delta^{n_2})$, for $n_1 \geq n_2 \geq 0, k \geq 0$, which becomes a *truncation polytope* if $n_2 = 0, 1$. From [3] we know that the corresponding moment-angle manifold is a connected sum of sphere products, as it is for a truncation polytope, but if we consider a vertex cut of a product of more than two simplices it may not be the case.
Following the ideas of [4],[1] and [5] the author obtained two main results stated as Theorem 1 and Theorem 2 below.

Theorem 1 *For a generalized truncation polytope P with $n_1 \geq n_2 > 1, k \geq 0$ the complex $K_P = \partial P^*$ is minimally non-Golod.*

Theorem 2 *For a generalized truncation polytope P with $n_1 \geq n_2 > 1, k \geq 0$ the bigraded Betti numbers of $\mathcal{Z}_P$ are given by the following formulae.*
If $n_1 = n_2 = n$, we have:

$$\beta^{-i,2(i+n)}(P) = 2\binom{k}{i-1}.$$

The work was partially supported by the Russian Foundation for Basic Research, grants 11-01-00694 and 12-01-92104

If $n_1 > n_2$, *we have:*

$$\beta^{-i,2(i+n_1)}(P) = \beta^{-i,2(i+n_2)}(P) = \binom{k}{i-1}.$$

$$\beta^{-i,2(i+1)}(P) = \beta^{-(k+2-i),2(k+1-i+n_1+n_2)}(P) = i\binom{k+2}{i+1} - \binom{k}{i-1}.$$

The other bigraded Betti numbers are ***zero****, except for*

$$\beta^{0,0}(P) = \beta^{-(m-d),2m}(P) = 1,$$

where m *is the number of facets of* P *and* d *is its dimension.*

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Factorization of Darboux transformations of arbitrary order for two-dimensional Schrödinger operator

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Nowadays the theory of exactly solvable one-dimensional and two-dimensional Schrödinger operators is inseparable from the theory of one- and two-dimensional integrable non-linear systems and the theory of Darboux-Bäcklund transformations (the soliton theory) [3, 4].

Here we prove (see a preliminary version at arxiv.org/abs/1304.7063) that a Darboux transformation of arbitrary order d for two-dimensional Schrödinger operator can be factored into Darboux transformations of order 1. By this, we close the problem on the structure of Darboux transformations for two-dimensional Schrödinger operators, which has been open for nearly 100 years.

Even for the special case of Darboux transformations of order 2 this problem is hard. For this case we have found earlier a (rather beautiful) proof [6] based on the invariantization (we used regularized moving frames due to Olver and Pohjanpelto).

The analogous statement for one-dimensional Schrödinger operator was proved in four steps (Shabat, Veselov [7, 5] and Bagrov, Samsonov [1, 2]). In this case the factorization is not unique, and different factorizations imply discrete symmetries related to the Yang-Baxter maps (Adler and Veselov).

The proof of the present result is based on some algebraic constructions which we use as a tool only. However, they probably have a more general meaning if reformulated in terms of abstract algebra.

The main result of the present paper implies that a Darboux transformation of arbitrary form and of arbitrary order d is invertible only in the case where it is a Laplace chain.

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On the product in negative Tate cohomology for compact Lie groups

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Classifying spaces of groups are objects of great importance in topology, geometry and algebra. Their homology and cohomology were studied for a long time, since they capture information about principal bundles and fibre bundles.

In this poster I will introduce a new product in the homology of classifying spaces of compact Lie groups. The grading of this product is given by:

$$H_k(BG,\mathbb{Z}) \otimes H_l(BG,\mathbb{Z}) \to H_{k+l+dim(G)+1}(BG,\mathbb{Z})$$

where $dim(G)$ is the dimension of the group G.

The construction is geometric and quite simple. For finite groups this product coincides with the cup product in negative Tate cohomology.

In the poster I will give some results regarding the product for connected groups, indicating the vanishing of this product for many groups, but not all. A part of the poster is a joint work with S. Kaji [3].

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On cohomological index of free G-spaces

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We will consider properties of cohomological index defined on a category of free G-spaces, where G is a finite group. For cyclic groups cohomological index was introduced by Yang and Bourgin via Smith sequences. For general groups it was constructed by Albert Schwarz (homological genus) and Conner & Floyd (cohomological co-index). Conner– Floyd's index $\mathrm{ind_L X}$ of a free G-space X is defined for any commutative ring with unit L. It can be shown that Schwarz's homological genus equals $\mathrm{ind_{\mathbb{Z}} X + 1}$, and that $\mathrm{ind_L X}$ satisfies all usual properties of Yang's homological index (defined for $G = \mathbb{Z}_2$) including the following property:

If $f\colon X \to Y$ is an equivariant map of free G-spaces and $\mathrm{ind_L X = ind_L Y = n}$, then $0 \neq f^*\colon H^n(Y; L) \to H^n(X; L)$.

In particular f is not homotopic to a constant map.

As a corollary we obtain:

1) Cohomological index is stable, i.e. $\mathrm{ind_L X * G = ind_L X + 1}$, where the join $X * G$ of X and G is considered with diagonal action of G.

2) If $f\colon X \to X$ is an equivariant selfmap then $0 \neq f^*\colon H^n(X; L) \to H^n(X; L)$, where $n = \mathrm{ind_L X}$. In particular, f is not homotopic to a constant map.

Second assertion has an overlap with a result of Gottlieb who showed that under some conditions Lefschetz number of f is divisible by the order of G and hence f is not homotopic to zero.

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We'll also consider other properties of index and applications to the space of circumscribed cubes around compact body in Euclidean space.

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